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Misconceptions about Density of Decimals: Insights from Indonesian Pre-service Teachers

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Extensive studies have documented various difficulties with, and misconceptions about, decimal numeration across different levels of education. This paper reports on pre-service teachers' misconceptions about the density of decimals. Written test data from 140 Indonesian pre-service teachers, observation of group and classroom discussions provided evidence of pre-service teachers' difficulties in grasping the density notion of decimals. This research was situated in a teacher education university in Yogyakarta, Indonesia. Incorrect analogies resulting from over generalization of knowledge about whole numbers and fractions were identified. Teaching ideas to resolve these difficulties and challenges in resolving pre-service teachers' misconceptions are discussed. Evidence from this research indicates that it is possible to remove misconceptions about density of decimals.

***Key words**:* Decimals; Pre-service teachers; Misconceptions; Density

Introduction

Difficulties about the teaching and learning of decimal numeration, including various misconceptions about decimals across different levels of education, have been well established (Brousseau, 1997; Brousseau, Brousseau, & Warfield, 2004, 2007; Stacey, 2005; Steinle, 2004). Misconceptions and difficulties with decimal numeration have also been observed in samples of pre-service teachers (Putt, 1995, Stacey, Helme, Steinle, Irwin, & Bana, 2001; Thipkong & Davis, 1991). Although Steinle and Stacey (2004) found that certain ways of thinking that are commonly observed in younger students, are infrequent in older students, teacher education studies indicate that some pre-service teachers hold misconceptions apparent in younger students. Steinle and Stacey concluded that the cumulative effect of instruction of many years is that some misconceptions are *covered over*, instead of *overcome*. The fact that pre-service teachers' misconceptions might be passed on to their future students provides an impetus for understanding and resolving these difficulties. In this paper, 'decimal' indicates any number written using a decimal point (or decimal comma in Indonesia).

Extensive studies have reported that pre-existing knowledge about whole numbers was often utilized utilised to interpret decimals (Hiebert, 1992; Moskal & Magone, 2000; Vamvakoussi & Vosniadou, 2004). The overgeneralisation of the discrete nature of whole numbers has been identified as one source of difficulty in grasping the properties of decimals. The continuity properties of real numbers (and hence of decimals) and the completeness of the real number line provide an example of this. These properties are manifested in several different ways. For example, given any decimal there is another arbitrarily close to it and all monotonically increasing sequences of decimals that are bounded above have a limit. In this paper, we are concerned only with one relatively simple version of the continuity properties, which we call the *density property*. This is the property that between any two decimals, there are infinitely many other decimals. This property is shared with the rational numbers – between any two fractions (rational numbers) there are an infinite number of fractions. It is not shared with the integers. For example, between 3 and 6 there are only two other integers (4 and 5). Hence the density of decimals is one example where pre-existing knowledge of whole numbers may lead to misconceptions about decimals.

This paper will report on pre-service teachers' thinking about density of decimals. The data reported in this paper is a small part of the larger study which aimed to study how to improve Indonesian pre-service teachers' content and pedagogical content knowledge on decimals. However, the instructional aspects are not the focus of this paper.

Literature Review

One of the features distinguishing decimals from whole numbers is the density of decimals. Empirical studies examining understanding of density involving children and adults have documented extensive difficulties in grasping this feature of decimals. Hiebert, Wearne, and Taber (1991) found that improving understanding of the continuity aspects of decimals was particularly difficult. Problems involving continuity properties, such as marking a decimal number on a number line or finding a number in between two given decimals such as 0.3 and 0.4, were found to be more challenging than problems involving discrete-representation task utilising multi-base arithmetic blocks (MAB). Analysis suggested that an extra step in finding the new 'unit' of the continuous models (e.g. thinking about hundredths, not just tenths in writing a number between 0.3 and 0.4) explained the lower performance on continuous-representation tasks.

Likewise, Merenluoto (2003) found concepts of density difficult for 16 to 17 year old students. She attributed difficulties with grasping density to students' reference to natural numbers and difficulties in extending their frame of reference to rational or real numbers. Furthermore, she contended that this kind of explanation was based on an abstraction from natural number properties rather than a radical conceptual change from natural to real numbers. Recent works of Merenluoto and Lehtinen (2004) and Vamvakoussi and Vosniadou (2004) used conceptual change perspective as an instructional strategy in overcoming students' difficulties with the density property. Vamvakoussi and Vosniadou (2004) identified several categories of understanding about density, which will be outlined in a later section.

Difficulties with density were also evident in studies involving pre-service teachers. Menon (2004) found only 59% of 142 pre-service teachers recognised the density of decimals. A similar trend was noted by Tsao (2005) who found that of 12 pre-service teachers involved in her study, only six high ability students demonstrated an understanding of density.

The nature of incorrect responses with regard to the density is reflected in common misconceptions drawing on analogies between decimals and whole numbers. In general, incorrect answers could be classified in two categories. Firstly, some students believe that there are no decimals between certain pairs of decimals. Fuglestad (1996) found that most students in her study of Norwegian students claimed there were no decimals in between two given decimals such as between 3.9 and 4 or between 0.63 and 0.64. Similarly, Bana, Farrell, and McIntosh (1997) reported that the majority of 12-year-olds and 14-year-olds from Australia, the United States, Taiwan and Sweden displayed the same problem. Only 62% of 14-year-olds from Australia and 78% of 14-year-olds from Taiwan showed understanding of decimal density. This evidence reflected incorrect extension of whole number knowledge that there is no whole number in between two consecutive whole numbers such as 63 and 64.

The second category of incorrect answer translates knowledge of multiplicative relations between subsequent decimal fractions of successive lengths. For instance, Hart (1981) reported that 22% to 39% of students age 12 to 15 years thought there were eight, nine, or ten decimals in between 0.41 and 0.42. They counted (correctly or incorrectly) the numbers 0.411, 0.412, …0.419. Tsao (2005) observed the same phenomenon in her study with pre-service teachers. She found that three pre-service teachers from a low ability group believed there were nine decimals in between 1.42 and 1.43 by counting only the thousandths: 1.421, 1.422,…, and 1.429.

As with the studies by Merenluoto and Lehtinen (2004) and Vamvakoussi and Vosniadou (2004), the present authors see these difficulties with decimals primarily as requiring conceptual change, rather than requiring adding knowledge of new facts. Some approaches to remedial education assume that students need to add knowledge to improve their understanding. However, as we have also argued in Pierce, Steinle, Stacey, and Widjaja (2008), additional learning or practice of procedural rules is often ineffective because students lack a correct conceptual foundation. For example, students could be taught to find some decimals between 0.3 and 0.4 by using a procedural rule: write 0.3 and 0.4 as 0.30 and 0.40 and then use whole number knowledge to insert the decimals in between for example, 0.31 and 0.32. However, this procedure is meaningless to students and therefore easily forgotten if they do not have a strong conceptual foundation for decimal notation. In contrast, conceptual teaching approaches such as the ones outlined in Bell (1993a, 1993b) have a strong record of effectiveness. The teaching approaches used in this paper followed these principles.

Taking into account the results from prior studies as discussed above, this paper will contribute to the discussion about the nature of Indonesian pre-service teachers' misconceptions about decimal density of decimals.

Methodology

Participants

The whole study comprised of two cycles and adopted a design research methodology (Brown, 1992; Gravemeijer, 1994), which cycles through design, teaching experiment, and retrospective analysis phases. This paper reports on a small portion of cycle 2 research data which were collected between August to November 2006. Some of the teaching approaches adopted had been refined since cycle 1. Others were newly introduced in cycle 2, to meet problems identified in the cycle 1. 140 pre-service teachers attending a private teacher training university in Yogyakarta from two cohorts fully participated in the cycle 2 study and completed both the pre-test and the post-test. Out of 140 pre-service teachers, 94 were enrolled in a two-year diploma programme run by the elementary teacher training department and the remaining 46 were enrolled in a Bachelor of Education (secondary) programme run by the science and mathematics education department.

Research Instruments

The data reported in this paper came from two pairs of parallel test items on the pre-test and post-test, and from episodes of group discussions pertinent to density of decimals. The pre-test and post-test items are shown in Figure 1 and translated from the original Indonesian version. Note that the pre-test and post-test items asked pre-service teachers to justify their choices. These explanations were envisaged to indicate or reveal pre-service teachers' thinking and misconceptions about density of decimals.

Insights into the participants' conceptions about density were also obtained from observing their strategies to find decimal numbers in between two given decimals. Relevant episodes from video recordings of group discussions in working with Activity 12 (see Figure 2) will be discussed in this paper to complement the written test data on their knowledge about density of decimals.

Pre-test Item 5
How many decimals can you find in between 3.14 and 3.15? Tick one of the options and explain briefly your reasoning.
- none, because ....
- 1, namely ....
- less than 200, because ....
- more than 200 but finite, because ....
- infinitely many, because ....

Post-test Item 5
How many decimals can you find in between 2.18 and 2.19? Tick one of the options and explain briefly your reasoning.
- none, because ....
- 1, namely ....
- less than 200, because ....
- more than 200 but finite, because ....
- infinitely many, because ....

Pre-test Item 6
How many decimals can you find in between 0.799 and 0.80? Tick one of the options and explain briefly your reasoning.
- none, because ....
- 1, namely ....
- less than 200, because ....
- more than 200 but finite, because ....
- infinitely many, because ....

Post-test Item 6
How many decimals can you find in between 0.899 and 0.90? Tick one of the options and explain briefly your reasoning.
- none, because ....
- 1, namely ....
- less than 200, because ....
- more than 200 but finite, because ....
- infinitely many, because ....

Figure 1. Pre-test and post-test items examining knowledge about density of decimals.

On both pre-test and post-test, items 5 and 6 were each scored out of 2. The score 0 was given for an incorrect answer or an answer that is correct but the example/explanation does not match. Score 2 was given for a correct answer with explanation. The total score for items 5 and 6 on the pre-test (and also the post-test) is therefore 0, 2 or 4.

12. For each pair of decimals in Table A, find decimals in between each pair of decimals if available. Justify your ways to find those decimals and give examples by locating them on the number line.

Table A

1.5	1.51
0.99	0.999
1.7501	1.75011

Figure 2. Activity 12 assessing strategies to find decimals in between two given decimals.

Results and Discussion

Findings from the Written Tests

Table 1 shows that many students had difficulty with ideas of density. Even though items 5 and 6 address slightly different cases (decimals of same and different lengths), the success rates of about a half the students in the pre-test for both items and about three-quarters in the post-test for both items show that these items behave similarly. Both primary and secondary cohorts recorded significant improvement on the items involving density of decimals, which indicated the positive impact of addressing the topic in the activities in this cycle. Table 2 shows the mean improvements and Table 1 shows that this was due to a drop in omissions as well as a reduction in incorrect answers. The gap between the mean scores of the primary and secondary cohorts was quite wide as can be observed in Table 2. This was expected as it is consistent with all other mathematics testing. The primary pre-service teachers recorded a high proportion of blank responses particularly in the pre-test and showed difficulties with density of decimals. Of all blank responses in pre-test items, only one came from the pre-service secondary teachers.

Table 1
Distribution of Responses to Density items in Pre-test and Post-test

Test	Item number	Number of students			
		Correct	Incorrect	Omitted	Total
Pre-test	5	74 (52.9%)	44 (31.4%)	22 (15.7%)	140 (100%)
	6	75 (53.6%)	45 (32.1%)	20 (14.3%)	140 (100%)
Post-test	5	102 (72.9%)	34 (24.3%)	4 (2.9%)	140 (100%)
	6	103 (73.6%)	31 (22.1%)	6 (4.3%)	140 (100%)

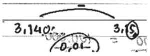
Table 2
Mean Scores on Pre- and Post-test on Density Items (Maximum possible score 4)

Cohorts	N	Pre-test		Post-test		Improvement	
		Mean	SD	Mean	SD	t-value	p-value
Primary	94	1.62	1.9	2.57	1.8	4.359	0.000
Secondary	46	3.17	1.5	3.65	1.0	2.119	0.040

Examining the pre-service teachers' responses to density items revealed four types of incorrect strategies and thinking. Some sample responses are given in Table 3. Some students' association of decimal numbers with whole numbers resulted in identifying that there were no decimals in between two given decimals in either item 5 or 6. The first line of Table 3 shows an example of a response in this category, as is the second line which is a little more sophisticated since it shows a decimals-fractions link. The third sample response in Table 3 shows that these students understand that there are some decimals in between the given ones, but they limit it to just one additional decimal place. This results in identifying a finite number of decimals in between two decimals, for example, noting that there are nine decimals in between 3.14 and 3.15. This strategy reflects the Indonesian curriculum sequence in teaching fractions and decimals at the primary schools, which encourages students to work with decimals of the same lengths only. The third set of incorrect responses showed a reliance on a "rounding rule" which was observed in finding the number of decimals in between 0.799 and 0.80. Stacey (2005) and Steinle and Stacey (2004) showed how excessive reliance on rounding is a feature of thinking of decimals as discrete, analogous to the way in which amounts of money in dollars and cents are discrete, with no allowable amounts between them. In the fourth set of incorrect responses, some pre-service teachers interpreted the question as subtraction to find the number of decimals in between two decimals (i.e.

the distance between). These unpredictable responses might indicate pre-service teachers' association of the words 'how many' or 'between' with finding differences between numbers, and possibly even difficulties in conceptualising 'numbers of numbers'. As illustrated in Table 3, the correct responses included some of the pre-service teachers giving examples only, some giving brief explanations and some citing known results on density.

Table 3
Sample Responses to Density Items

Test item	Sample responses	
	Incorrect	Correct
Item 5. (Pre-test or post-test)	• There are no decimals in between 3.14, and 3.15 because 14 and 15 are consecutive numbers. • There are no decimals in between 3.14, and 3.15 because $3.14 = 3\frac{14}{100}$ and $3.15 = 3\frac{15}{100}$ and there is no number in between $\frac{14}{100}$ and $\frac{15}{100}$. • Less than 200, there are 9, e.g., 3.141, 3.142,..., and 3.149. • 1, namely 0.01.  	• In between 3.14 and 3.15 there are more than 5 decimals, for example 3.142 and between 3.14 and 3.142 there are more than 5 decimals and we can continue like this forever so there are infinitely many in between 3.14 and 3.15. • Decimals can be converted to fractions and based on theory there are infinite numbers between two fractions. • Because decimals are subset of real numbers, it satisfies the dense property of real numbers.
Item 6. (Pre-test or post-test)	• There are no decimals in between 0.799 and 0.80 because 0.80 is the result of rounding of 0.799. • 1, namely 0.0001.	• There are infinitely many decimals that is larger than 0.799 smaller than 0.80, e.g. 0.799123, 0.7990001, etc. • Infinitely many numbers in between 0.899 and 0.90, for example 0.899001, 0.89990001, etc.

Insights from Group Discussions about Density of Decimals

In this section, episodes from a video-recording of two group discussions from the primary cohort will be presented to give additional insights into the participants' knowledge about density of decimals. These groups employed a strategy based on (sometimes successive) partitioning into ten of the interval between the given pair of decimals. Sari, Aris, and Bayu were pre-service primary teachers who worked together as a group during the teaching experiment, whilst Rori worked together with three other pre-service teachers in a different group.

> Sari: So the number of numbers in between these two numbers is infinite.
>
> Aris: But the way we find them is by first partitioning into ten then we find that there are infinitely many numbers.
>
> Bayu: So first, we divide the interval into ten equal parts.
>
> Aris: Yes, we divide the interval into ten equal parts so the conclusion there are infinitely many.
>
> Sari: You need to write that down… so first we divide the interval between every pair of numbers into ten then after dividing into ten parts we know that we can continue divide the interval into ten parts. So the conclusion is there are infinitely many numbers.

Note that Sari's group, consisting of Sari, Aris and Bayu employed a strategy that related to the second strategy, that is, working with decimals with the same length. However, Sari's group was able to extend the thinking to involve beyond decimals only with the same length or with one length greater. Hence Sari's group was able to 'see' that there are infinitely many decimals in between any given decimals.

As with Sari's group, Rori's group also worked with decimals with the same lengths and partitioned them successively into ten equal sections. Their written work is shown in Figure 3. Note that in the Indonesian context, a comma is used instead of a point to signify decimal numbers. Rori's group first converted the given decimals into corresponding equivalent fractions and found fractions with this denominator in between. Then they used a different common denominator. Counting only the number of thousandths, Rori's group found there were eight 'thousandths numbers' in between $\frac{990}{1000}$ and $\frac{999}{1000}$ whilst counting only ten thousandths, there were 89 'ten thousandths numbers' in between $\frac{9900}{10000}$ and $\frac{9990}{10000}$. As shown in Figure 3, Rori's group found two different numbers of decimals in between 0.99 and 0.999 but could not decide if the answer was 8 or 89 decimals.

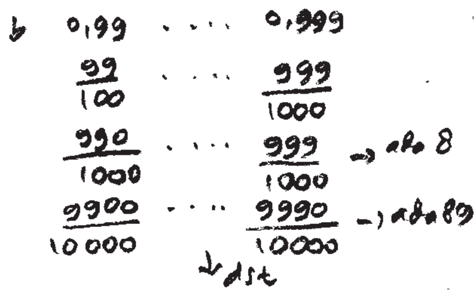


Figure 3. Numbers in between 0.99 and 0.999 taken from Rori's group worksheet.

Even though the strategy used by Rori's group was more sophisticated than some others seen, for example in Table 3, it still showed a tendency to work only with decimals and fractions from 'the same worlds' which all have the same digit-length (i.e. the same denominators as fractions). Moreover, this strategy inhibited Rori's group from perceiving that there are infinitely many decimals in between 0.99 and 0.999. A similar trend was observed and reported by Merenluoto and Lehtinen (2002) in the following quote:

> Even at the higher levels of education, students seem to be unaware of their thinking about numbers or the fundamental difference between natural and rational numbers. Because of the operational justification of the extension of number concept, little attention is paid to the underlying general principles of the different number domains in the curriculum. (p.522)

Rori's strategy could be classified as 'discreteness-density' in the scheme proposed by Vamvakoussi and Vosniadou (2004). They noted that students who belong to the discreteness-density category "give seemingly inconsistent answers, in the sense that they do not answer in the same way questions concerning decimals with the same number of digits…" (p.464) and they justify their inconsistent answers based on "different groups of numbers" (p.465).

Conclusion and Implications

Findings from this study showed that understanding the density property of decimals was a challenging task for these pre-service teachers, which was evident from the written tests and the group discussions in the teaching experiment. Mathematical textbooks often provide exercises where students need to work only with decimals with the same number of decimal places to avoid complications. This appears to constrain students from appreciating the continuity properties of decimals including its density property. Although this paper was not focused on the teaching intervention, it was pleasing to see that both cohorts improved significantly, even though a quarter of the cohort still had incorrect answers. The conceptual change perspective that was employed has been advocated by many authors, including Bell (1993a, 1993b), McIntosh, Stacey, Tromp, & Lightfoot (2000), Merenluoto & Lehtinen (2004), Pierce, Steinle, Stacey, & Widjaja (2008) and Vamvakoussi & Vosniadou (2004). These studies highlighted the need for fundamental reorganisation of prior knowledge of whole numbers in understanding the density of decimals and also of rational numbers. Our study concur with prior research that understanding of density of decimals is not easy and the discreteness feature of whole numbers is inapplicable for understanding density nature of decimals.

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